1. The length of the diagonal of square S, as well as the lengths of the diagonals of rhombus R are integers. The ratio of the lengths of the diagonals is 15:11:9, respectively. Which of the following could be the difference between the area of square S and the area of rhombus R?

I. 63
II. 126
III. 252

A. I only
B. II only
C. III only
D. I and III only
E. I, II and III

Given that the ratio of the diagonal is $d_s:d_1:d_2=15x:11x:9x$, for some positive integer x (where d_s is the diagonal of square S and d_1 and d_2 are the diagonals of rhombus R).

$area_{square}=\frac{d^2}{2}$ and $area_{rhombus}=\frac{d_1*d_2}{2}$.

The difference is $area_{square}-area_{rhombus}=\frac{(15x)^2}{2}-\frac{11x*9x}{2}=63x^2$.

If x=1, then the difference is 63;
If x=2, then the difference is 252;
In order the difference to be 126 x should be $\sqrt{2}$, which is not possible.

Answer: D.

2. Set S contains 7 different letters. How many subsets of set S, including an empty set, contain at most 3 letters?

A. 29
B. 56
C. 57
D. 63
E. 64

1 empty set;
$C^1_7=7$ sets with one element;
$C^2_7=21$ sets with two elements;
$C^3_7=35$ sets with three element.

Total 1+7+21+35=64 sets.

Answer: E.

3. How many different subsets of the set {0, 1, 2, 3, 4, 5} do not contain 0?

A. 16
B. 27
C. 31
D. 32
E. 64

Consider the set without 0: {1, 2, 3, 4, 5}. Each out of 5 elements of the set {1, 2, 3, 4, 5} has TWO options: either to be included in the subset or not, so total number of subsets of this set is 2^5=32. Now, each such set will be a subset of {0, 1, 2, 3, 4, 5} and won't include 0.

Answer: D.

4. The functions f and g are defined for all the positive integers n by the following rule: f(n) is the number of perfect squares less than n and g(n) is the number of primes numbers less than n. If f(x) + g(x) = 16, then x is in the range:

A. 30 < x < 36
B. 30 < x < 37
C. 31 < x < 37
D. 31 < x < 38
E. 32 < x < 38

Perfect squares: **1, 4, 9, 16, 25,** 36, ..,
Prime numbers: **2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,** 37, ...

If x = 31, then f(31) = 5 and g(31) = 10: f(x) + g(x) = 5 + 10 = 15.
If x = 32, then f(32) = 5 and g(32) = 11: f(x) + g(x) = 5 + 11 = **16**.
...
If x = 36, then f(36) = 5 and g(36) = 11: f(x) + g(x) = 5 + 11 = **16**.
If x = 37, then f(37) = 6 and g(37) = 11: f(x) + g(x) = 6 + 11 = 17.

Thus x could be 32, 33, 34, 35 or 36: 31<x<37.

Answer: C.

5. Which of the following is a factor of 18!+1?

A. 15
B. 17
C. 19
D. 33
E. 39

18! and 18!+1 are consecutive integers. Two consecutive integers are co-prime, which means that they don't share ANY common factor but 1. For example 20 and 21 are consecutive integers, thus only common factor they share is 1.

Now, since we can factor out each 15, 17, 33=3*11, and 39=3*13 out of 18!, then 15, 17, 33 and 39 ARE factors of 18! and are NOT factors of 18!+1. Therefore only 19 could be a factor of 18!+1.

Answer: C.

6. If the least common multiple of a positive integer x, 4^3 and 6^5 is 6^6. Then x can take how many values?

A. 1
B. 6
C. 7
D. 30
E. 36

We are given that $6^6=2^6*3^6$ is the least common multiple of the following three numbers:

x;
$4^3=2^6$;
$6^5=2^5*3^5$;

First notice that x **cannot have any other primes other than 2 or/and 3**, because LCM contains only these primes.

Now, since the power of 3 in LCM is higher than the powers of 3 in either the second number or in the third, than x must have 3^6 as its multiple (else how 3^6would appear in LCM?).

Next, x **can have 2 as its prime in ANY power ranging from 0 to 6, inclusive** (it cannot have higher power of 2 since LCM limits the power of 2 to 6).

Thus, x could take total of 7 values.

Answer: C.

7. The greatest common divisor of two positive integers is 25. If the sum of the integers is 350, then how many such pairs are possible?

A. 1
B. 2
C. 3
D. 4
E. 5

We are told that the greatest common factor of two integers is 25. So, these integers are $25x$ and $25y$, for some positive integers x and y. Notice that x and y must not share any common factor but 1, because if they do, then GCF of $25x$ and $25y$ will be more that 25.

Next, we know that $25x+25y=350$ --> $x+y=14$ --> since x and y don't share any common factor but 1 then (x, y) can be only (1, 13), (3, 11) or (5, 9) (all other pairs (2, 12), (4, 10), (6, 8) and (7, 7) do share common factor greater than 1).

So, there are only three pairs of such numbers possible:
25*1=25 and 25*13=325;
25*3=75 and 25*11=275;
25*5=125 and 25*9=225.

Answer: C.

8. The product of a positive integer x and 377,910 is divisible by 3,300, then the least value of x is:

A. 10
B. 11
C. 55
D. 110
E. 330

Given: $\frac{377,910*x}{3,300}=integer$.

Factorize the divisor: $3,300=2^2*3*5^2*11$.

Check 377,910 for divisibility by 2^2: 377,910 IS divisible by 2 and NOT divisible by 2^2=4 (since its last two digits, 10, is not divisible by 4). Thus x must have 2 as its factor (377,910 is divisible only by 2 so in order 377,910*x to be divisible by 2^2, x must have 2 as its factor);

Check 377,910 for divisibility by 3: 3+7+7+9+1+0=27, thus 377,910 IS divisible by 3.

Check 377,910 for divisibility by 5^2: 377,910 IS divisible by 5 and NOT divisible by 25 (in order a number to be divisible by 25 its last two digits must be 00, 25, 50, or 75, so 377,910 is NOT divisible by 25). Thus x must have 5 as its factor.

Check 377,910 for divisibility by 11: (7+9+0)-(3+7+1)=5, so 377,910 is NOT divisible by 11, thus x must have 11 as its factor.

Therefore the least value of x is $2*5*11=110$.

Answer: D.

9. What is the 101st digit after the decimal point in the decimal representation of 1/3 + 1/9 + 1/27 + 1/37?

A. 0
B. 1
C. 5
D. 7
E. 8

.

102nd digit will be 8, thus 101st digit will be 0.

Answer: A.

10. If x is not equal to 0 and x^y=1, then which of the following must be true?

I. x=1
II. x=1 and y=0
III. x=1 or y=0

A. I only
B. II only
C. III only
D. I and III only
E. None

Notice that if x=-1 and y is any even number, then $(-1)^{even}=1$, thus none of the options must be true.

Answer: E.